Ocean City High School

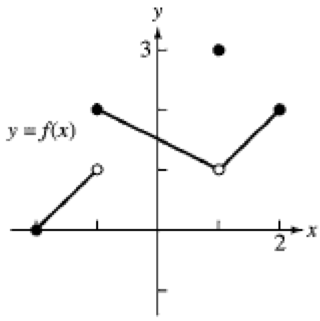
Summer Assignment 2022

Course	AP Calculus BC	**Teacher**	Dill
Email	mdill@ocsdnj.org	**Due Date**	August 15^{th}, 2022
Standards			
Topic	Limits, continuity, derivatives, applications of derivatives, integrals and applications of integrals.		
Purpose	The purpose of this assignment is to refresh you on the topics from last year. We will not spend time in the beginning of AP Calc BC reviewing topics from last year. You are expected to refresh yourself on those topics on your own time.		
Text/Novel(s) & Brief Description	There are two pages on each topic. The first page is required and the second page is provided for optional extra practice.		
Approximate Time on Task	This assignment should take you about 2-3 hours.		
Suggested Timeline	I suggest you begin the assignment in mid July and work a little at a time until early August.		
How It Will Be Assessed	**Assignment is to be turned in to me by 8/15 either by dropping off at school into my mailbox or by sending it through the mail**. The assignment will be graded out of 30 points (15 for completion and 15 points for accuracy).		

At the suggestion of the students, I created an assignment that will refresh you on the topics from AP Calculus AB. I understand that some of you need more practice in certain areas and that can vary from student to student. For this reason, I provided two pages on each topic. The first page is required and the second page on each topic is for extra practice if you do not yet feel refreshed after the first page. Please email me with questions or concerns.

Limits and Continuity

1) For the function $y = f(x)$ whose graph is shown below, which statement is false?



(A) $\lim_{x \to 1} f(x) = 1$

(B) $\lim_{x \to 2^-} f(x) = 2$

(C) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(D) $\lim_{x \to -1} f(x) = 2$

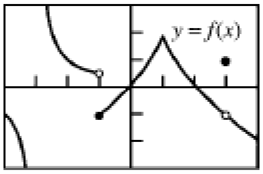
(E) $\lim_{x \to -1^+} f(x) = 2$

2) Let $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ -\frac{1}{2}x + 1, & x \geq 1 \end{cases}$. What is $\lim_{x \to 1^+} f(x)$?

(A) -1 (B) $\frac{1}{2}$ (C) 1

(D) 1.73 (E) Does not exist

3) The function f whose graph is shown below is continuous at which of the following points?



[−4, 4] by [−3, 3]

(A) $x = -3$ (B) $x = -1$ (C) $x = 1$

(D) $x = 3$ (E) All of these.

4) Find the average rate of change of the function $f(x) = 100 - 16x^2$ over the interval [0, 2].

(A) -64 (B) -36 (C) -32 (D) 32 (E) 36

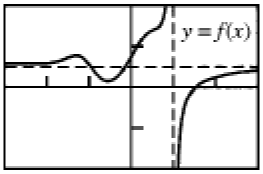
5) Consider the function $f(x)$ given below. Which of the following appear to be true about $f(x)$?

I. The line $y = \frac{1}{2}$ is a horizontal asymptote.

II. $\lim_{x \to 2} f(x) = 2$

III. The line $x = 1$ is a vertical asymptote.

IV. $\lim_{x \to +\infty} f(x) = \lim_{x \to -\infty} f(x)$



[−3, 3] by [−2, 2]

(A) I and III
(B) III and IV
(C) I, II, and III
(D) I, III, and IV
(E) I, II, III, and IV

6) Find a value a so that the function

$$f(x) = \begin{cases} 5 - ax^2, & x < 1 \\ 4 + 3x, & x \geq 1 \end{cases}$$ is continuous.

7) Sketch a possible graph for a function f, where $f(-2)$ exists, $\lim_{x \to -2} f(x) = 2$, and f is not continuous at $x = -2$.

8)

Let $f(x) = \begin{cases} x^2 + 5, & x \leq 2 \\ \dfrac{4x - 3}{x + 3} & x > 2 \end{cases}$.

Find the limit of $f(x)$ as **(a)** $x \to -\infty$, **(b)** $x \to 2^-$, **(c)** $x \to 2^+$, and **(d)** $x \to \infty$

Derivatives

1)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

(a) $3f(x) - g(x), \quad x = -1$

(b) $f^2(x)g^3(x), \quad x = 0$

(c) $g(f(x)), \quad x = -1$

(d) $f(g(x)), \quad x = -1$

(e) $\dfrac{f(x)}{g(x) + 2}, \quad x = 0$

(f) $g(x + f(x)), \quad x = 0$

For number 2-4, find the derivative.

2) $y = \ln(\sin x)$

3) $\sin(x + y) = \tan(xy)$

4) $y = x^2 \tan(3x - 1)$

5) ***Vertical Motion*** On Earth, if you shoot a paper clip 64 ft straight up into the air with a rubber band, the paper clip will be $s(t) = 64t - 16t^2$ feet above your hand at t sec after firing.

(a) Find ds/dt and d^2s/dt^2.

(b) How long does it take the paper clip to reach its maximum height?

(c) With what velocity does it leave your hand?

(d) On the moon, the same force will send the paper clip to a height of $s(t) = 64t - 2.6t^2$ ft in t sec. About how long will it take the paper clip to reach its maximum height, and how high will it go?

6)

Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} (3t^2 - 5t)\, dt$.

(A) $3x^2 - 5x$ (B) $6x^3 - 10x^2$ (C) $12x^3 - 10x$
(D) $6x^5 - 10x^3$ (E) $3x^6 - 5x^4$

7) Find $\frac{d^2y}{dx^2}$ if $x^2 + 3y^2 = 10$

8)

Find $\frac{dy}{dx}$, where $y = \frac{x^2 - 5}{\cos x}$.

9) A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 12t + 5$.

(a) Find the velocity of the particle at any time t.

(b) Find the acceleration of the particle at any time t.

(c) Find all values of t for which the particle is at rest.

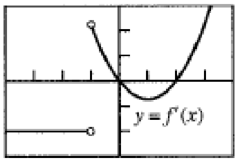
(d) Find the speed of the particle when its acceleration is zero.

(e) Is the particle moving toward the origin or away from the origin when $t = 3$? Justify your answer.

Applications of Derivatives

1)

The graph below represents the *derivative* of a function f, where $f(-1) = -2$. Sketch a possible graph of $y = f(x)$.



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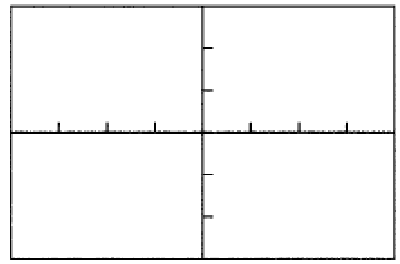
6.



[–4, 4] by [–3, 3]

2)

Use graphing techniques to determine the interval(s) where the graph of the function $y = x - 3e^{-x^2}$ is

(a) increasing, **(b)** decreasing,

(c) concave up, **(d)** concave down.

Then find any

(e) local extreme values, **(f)** inflection points.

3)

Sally makes an open box from a 30 inch by 42 inch rectangular piece of cardboard by cutting congruent squares from each of the corners and folding up the sides.

(a) What size square should she cut from each corner so that the box will have maximum volume?

(b) What is the maximum volume for the box?

4)

Let V be the volume of a closed rectangular box whose edges have lengths x, y, and z. Assume that x, y, and z vary with time. If x increases at the constant rate of 4 ft/sec, z decreases at the constant rate of -3 ft/sec, and y is unchanged, how fast is the volume changing when $x = 10$ feet, $y = 8$ feet, and $z = 5$ feet?

5)

Find the linearization $L(x)$ of $f(x) = 3x^4 - 5x^3$ at $x = 2$.

Use it to estimate f(2.1).

6)

Area of Triangle An isosceles triangle has its vertex at the origin and its base parallel to the x-axis with the vertices above the axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.

7) ***Changing Cube*** The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the edges changing at that instant?

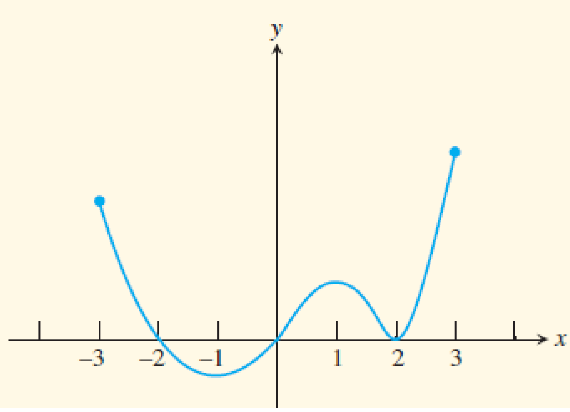
8) The accompanying figure shows the graph of the derivative of a function f. The domain of f is the closed interval $[-3, 3]$.

(a) For what values of x in the open interval $(-3, 3)$ does f have a relative maximum? Justify your answer.

(b) For what values of x in the open interval $(-3, 3)$ does f have a relative minimum? Justify your answer.

(c) For what values of x is the graph of f concave up? Justify your answer.

(d) Suppose $f(-3) = 0$. Sketch a possible graph of f on the domain $[-3, 3]$.



Integrals

1) $\int 3x^2\, g'(x^3)\, dx$

2) $\int \frac{6x}{\left(3x^2-7\right)^4}\, dx$

3) $\int \sin x \cos x\, dx$

4) $\int \sin(2x)\, dx$

5) $\int \sin\theta\, (\cos\theta + 5)^7\, d\theta$

6) $\int \frac{5}{\sqrt{5x-1}} \, dx$

7) $\int 3x^2 \cos\left(x^3\right) dx$

8) $\int 2\sqrt{2x+1} \, dx$

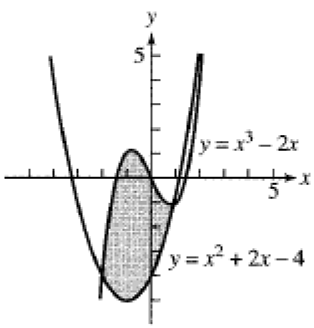
9) $\int 2t \sin\left(t^2\right) dt$

20) $\int 2\, g'(2x) \, dx$

Applications of Integrals

1) At a certain paint factory, the production rate in gallons per hour x hours after the factory opens in the morning is given by the function $r(x) = -\frac{3}{5}x^4 + 16x^3 - 135x^2 + 400x$. How much paint is produced during the first 3 hours after opening?

2) What is the area of the shaded region on the graph shown?



3) A region is bounded by the line $y = x$ and the parabola $y = x^2 - 6x + 10$. Find the volume of the solid generated by revolving the region about the x-axis.

4) The table below shows the velocity of a car in an amusement park ride during the first 24 seconds of the ride. Use the left-endpoint values (LRAM) to estimate the distance traveled, using 6 intervals of length 4.

Time (sec)	0	4	8	12	16	20	24
Velocity (ft/sec)	0	4	7	7	11	16	18

5)

Multiple Choice The base of a solid is the region in the first quadrant bounded by the x-axis, the graph of $y = \sin^{-1} x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume?

(A) 0.117 **(B)** 0.285 **(C)** 0.467 **(D)** 0.571 **(E)** 1.571

6)

Multiple Choice A developing country consumes oil at a rate given by $r(t) = 20e^{0.2t}$ million barrels per year, where t is time measured in years, for $0 \le t \le 10$. Which of the following expressions gives the amount of oil consumed by the country during the time interval $0 \le t \le 10$?

(A) $r(10)$

(B) $r(10) - r(0)$

(C) $\int_0^{10} r'(t)\,dt$

(D) $\int_0^{10} r(t)\,dt$

(E) $10 \cdot r(10)$

7)

Free Response Let R be the region bounded by the graphs of $y = \sqrt{x}$, $y = e^{-x}$, and the y-axis.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = -1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle whose diameter runs from the graph of $y = \sqrt{x}$ to the graph of $y = e^{-x}$. Find the volume of this solid.